

# How High in Frequency Are Accelerometer Measurements Meaningful?

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Almost all piezoelectric accelerometers in the current market place have a fundamental sensor resonance below 100 KHz. In 1983 Endevco Corporation designed a series of MEMS (Micro Electro-Mechanical Systems) accelerometers [1]. These silicon-based piezoresistive accelerometers enabled sensor resonances of 100s of KHz to above 1 MHz. The original intent of this design (Model 7270) was to create an accelerometer with a resonant frequency high enough that it would not be excited in metal-to-metal impact or explosive environments. These types of environments are generally described by the term pyrotechnic shock (pyroshock). Unfortunately, in spite of the advantage provided by the high resonant frequencies of these accelerometers, the extremely low intrinsic damping of silicon acts as a counterbalance. The result of this low damping is often over-ranging and breakage of the accelerometers when they are subjected to pyroshock [1]. In order to support the development of a series of more robust MEMS accelerometers, this paper answers the question: How high in frequency response are accelerometer measurements meaningful?

## INTRODUCTION

In 2008, a new MEMS shock accelerometer (PCB Piezotronics Model 3991) was designed and introduced into the market place. While this accelerometer has the same foot print as the Endevco 7270, its most significant differences are its contained damping ( $\sim 0.05$  of critical viscous,  $Q=10$ ) and over-range stops [2]. The goal of the damping is to reduce the “Q” of the sensor at its resonant frequency. This lowered “Q” should lessen the fragility and associated over-range issues associated with MEMS accelerometers in high frequency (e. g., pyroshock) environments. To enable damping to be achieved, the compliance of the sensing element of the Model 3991 had to be increased (i. e., its stiffness had to be decreased). This initiated the discussion, which is the subject of this paper: *How high in frequency are accelerometer measurements meaningful?* The author contributed to this discussion by opining that 20 KHz was a realistic upper limit. This paper provides the basis for this opinion.

Actually, the author had encountered this discussion twice before. In the early 1970s, at a Transducer Workshop sponsored by the DoD National Test Ranges, a need was expressed for accelerometers capable of measuring to 100 KHz to support shock wave physics experiments. At that time, no accelerometer capable of this measurement could even be conceptualized. In 1976, at Sandia National Laboratories, the request for an extremely high frequency acceleration measurement was again presented to the author. The goal was to characterize the structural input to hypersonic vehicles flying through rain storms. This input was theorized to have a power spectral density (PSD) surpassing 50 KHz in frequency content. The author performed several calculations that discouraged any attempt at measurements to support a test series.

In the ensuing years, several standards have evolved to define and support pyrotechnic shock testing [3]. These include: (1) IEST-RP-DTE032.1, Pyroshock Testing Techniques (being reviewed and updated), (2) MIL-STD-810F, Method 517 (changes currently being proposed), and (3) NASA HDBK-7003. In these standards are definitions for near field pyroshock. Respectively, in terms of amplitude and frequency content, these definitions are:

IEEST-RP-DTE032.1: no magnitude specified, > 10,000 Hz in frequency content,  
MIL-STD-810F: > 5,000 G in magnitude, > 100,000 Hz in frequency content, and  
NASA HDBK-7003: > 5,000 G in magnitude, > 100,000 Hz in frequency content.

These definitions might imply that frequencies much higher than 10,000 Hz can and should be measured by surface mounted accelerometers. This implication would be incorrect and is supported by the following four (4) arguments:

1. Calibration limitations in the national system of standards preclude the verification of the performance of accelerometers above 20 KHz.
2. Structural modeling of a unit under test (UUT) to very high frequencies typically lacks adequate predictive capabilities to correlate with experimental structural measurements.
3. The physical size of accelerometers tends to provide spatial averaging of the structural response of the UUT at high frequencies.
4. At very high frequencies, the physical presence of the accelerometer modifies the response of the structure to which it is affixed.

Each of these arguments will be investigated and individually supported.

### **BODY**

Figure 1 shows a PCB Model 3991 (geometrically identical to an Endevco 7270) sitting on top of a United States one-cent piece. The coin's dimensions are 0.75 inches or 19 mm in diameter and 0.05 inches or 1.27 mm in

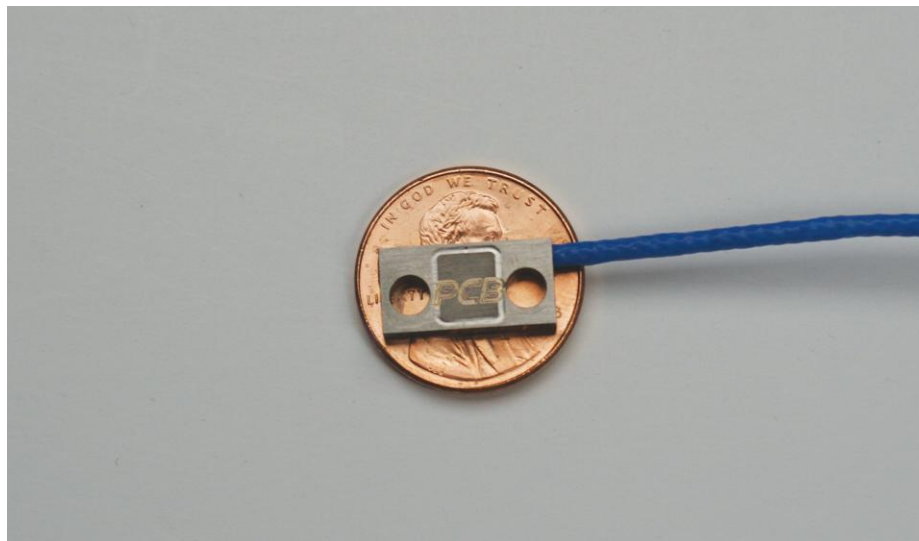


Figure 1: PCB Model 3991 on a United States One-Cent Coin (penny)

thickness. The resonant frequency for the first mode of vibration of the one-cent coin alone, with free edges, can be calculated to be 12,470 Hz. Its mass is approximately 2.57 grams. Envision how radically this resonant frequency would decrease due to the stiffening effect and the addition of the 1.28 gram mass of the accelerometer if it were physically attached to the coin! This observation supports the premise of this paper. That is, there is an upper frequency limit above which an accelerometer stops providing a meaningful structural response assessment of the UUT. The challenge then is to define this limit.

#### **Calibration Limitations:**

Most developed countries have a centralized, government-supported, calibration service. For the United States of America this service is the National Institute of Standards and Technology (NIST). At NIST, the upper frequency limit of accelerometer calibration services is 20,000 Hz [4]. Specifically, this calibration is performed from 3 KHz

to 20 KHz at constant displacement amplitude of 121.10 nm, which enables its accurate measurement. The resultant acceleration varies from about 4 G at 3 KHz to 200 G at 20 KHz. Measurement uncertainty is 1 – 3 %.

Above 20 KHz the air bearing shakers [5], typically used to provide linear motion, begin to become operationally limited. In addition, at any constant G-level, as vibration frequencies increase their associated displacements decrease, and the accuracy of laser based interferometer measurements is lessened. These interferometer measurements, along with independent frequency determinations, provide the basis for the calibration. In summary, 20 KHz represents the upper frequency limit for quantitative accelerometer calibrations both nationally and internationally.

Still focusing on the calibration issue, it should be noted that MEMS piezoresistive accelerometers typically provide between 100 and 200 mV full-scale output. Thus, a 20,000-G accelerometer, typical of those used for pyroshock measurements, would provide signal levels between 0.02 and 0.04 mV at 4 G and between 1 and 2 mV at 200 G. These low signal levels further increase the uncertainty in the determination of the frequency-response function for an accelerometer (See Fig. 2). The red curve is the most probable response of the Model 3991 and the actual test data (blue scatter) contains uncertainty due to its low signal output. Thus, frequency-response determinations become further complicated by low signal levels from the accelerometer under test. This provides additional credibility for limiting the performance certification of accelerometers to an upper frequency of 20 KHz. Above that value, unanticipated torsional or lateral resonances internal to the accelerometer's sensing element [6], resonances in the accelerometer's case, and lead-wire attachment resonances can erroneously be interpreted as structural response of the UUT. None of these resonances would have been identified in the calibration process. Mounting anomalies can also be a further error contributor.

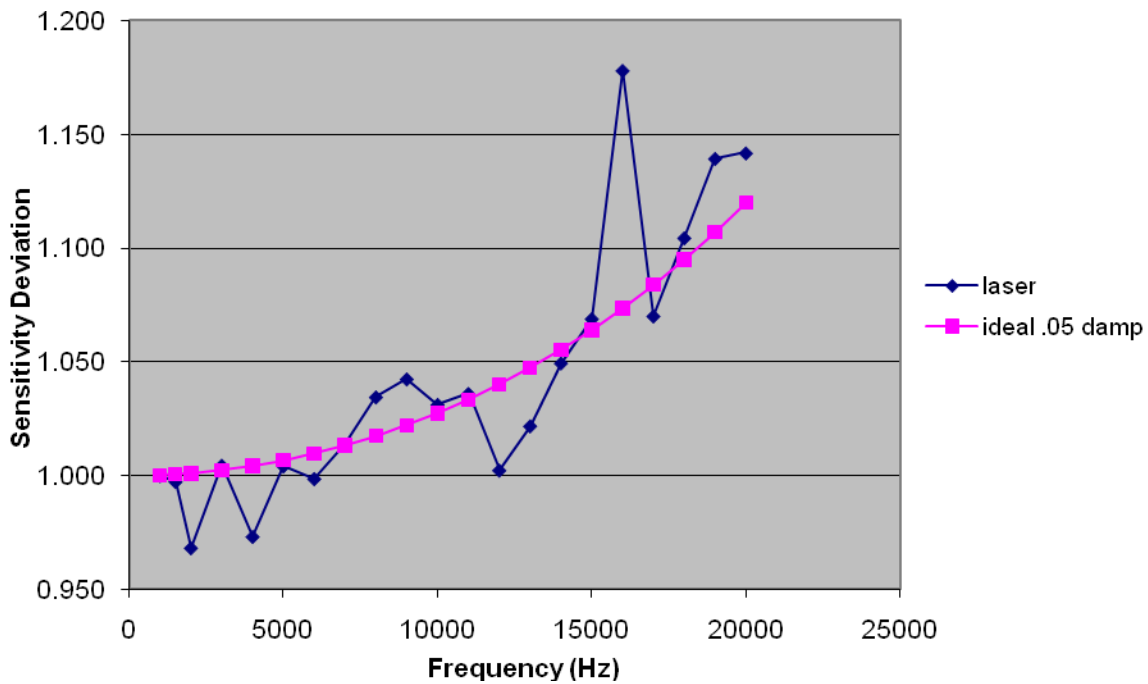


Figure 2: Vibration Test Results on a Model 3991

#### Modeling Limitations at High Frequencies:

Pyroshock testing techniques first evolved in support of the aerospace community. Most UUTs associated with this community are fairly substantial in size (e.g., satellites with volumes as small as a few cubic feet up through large launch vehicles). As a byproduct, their fundamental resonant frequencies are low. However, high frequencies

become of interest when assessing the response to pyroshock stimuli of smaller, contained electronics or other fragile items internal to the UUT.

Modeling the response of complex structures to high frequencies necessitates their discretization into a very large number of elements. This number of elements can be thousands, tens of thousands, or even higher. For a given structure, as the number gets larger, the element size progressively decreases. Even though the UUT may be substantial in size, the accelerometer measures the response of the element to which it is affixed. Thus, as we attempt to make an experimental correlation of structural dynamics measurements to predictive models at high frequencies, we reduce the problem to the dilemma presented in Fig 1.

One question that could be asked is: Why not include the structural characteristics of the accelerometer in the modeling? The answer is that these characteristics (particularly those of the interconnecting cable) are not well defined, and the mounting attachment can also be a variable. For example, an article in *Machine Design* (11/15/2002) notes that there are 75 factors that affect the tension in applying torque to a single bolt.

While this discussion of modeling limitations has not produced a definitive upper frequency limit for meaningful accelerometer measurements, it is consistent with prior discussions that such a limit must exist.

#### Spatial Averaging Considerations of the Measurements:

Digital sampling of data requires at least two samples per cycle of a sine wave to preserve its frequency content. This requirement is based on the well-known Shannon theorem, and sampling that satisfies it is referred to as Nyquist sampling. However, if the goal of the measurement is to obtain an experimental peak value of a sinusoidal signal at that frequency, sampling must occur at not less than ten equally spaced intervals per cycle. This sampling density assures no more than a 5% peak error. This last requirement enables one to infer that the wavelength of any traveling elastic wave is particularly important in establishing the minimum accelerometer spatial mounting density required to define the upper frequency content of pyroshock or other high-frequency events. This wavelength must then contribute to defining an upper frequency limit for meaningful measurements.

The velocity of propagation of elastic waves traveling in a solid is defined by the Lamé constants [7]  $\lambda$  and  $\mu$ . Assuming an isotropic, unbounded solid medium, the surface of a discontinuity advances through the solid with a velocity of:

$$\sqrt{((\lambda+2\mu)/\rho)} \quad (1)$$

for waves of dilation and

$$\sqrt{(\mu/\rho)} \quad (2)$$

for waves of distortion. All plane waves travel with the velocities obtained in either Eqs. (1) or (2) above. A third type wave (Raleigh) also exists and propagates on the surface of a solid, elastic body. Its velocity is always between the two above, so the dilational and distortional waves will be evaluated as limiting cases. The Lamé constants are defined by the material properties E (Young's modulus) and  $\nu$  (Poisson's ratio). Steel (E = 30,000,000 psi,  $\nu = .33$ ) will be selected as a material to perform some representative calculations. Steel results in a calculated nominal velocity for the dilation wave (P-wave) of 245,000 inches/second. Its distortional wave (S-wave) has a calculated nominal velocity of 123,000 inches/second. Again, the Raleigh wave has an intermediate velocity. Using the relationship:

$$(\text{wavelength}) \times (\text{frequency}) = \text{velocity} \quad (3)$$

we can find the corresponding wavelengths for the P and S waves in steel at 20 KHz. These calculate to be nominally 12 inches for the P-wave and 6 inches for the S-wave.

One-inch is a typical minimum spacing between accelerometers mounted on a structure. Using the just-presented requirement (in the two underlined sentences just above Eq. (1)) for 10 samples/cycle to define the peak value of a sine wave within 5%, the above wavelength values (6 and 12 inches) also indicate 20 KHz to approximate a nominal

upper limit at which one should expect to acquire quantitative structural dynamics measurements with accelerometers. (Note: This observation is based on the fact that 10 samples/cycle at 1-inch intervals enable the accurate definition of a wavelength 10 inches long.)

Modification of the Structural Response by the Accelerometer:

Mechanical impedance can be considered to be the resistance of a structure to motion. If  $F$  is the peak harmonic input force to a structure, and  $V$  is the velocity response at a point, then the mechanical impedance at a given frequency is:

$$Z_{\text{mech}} = (F/V)e^{j\theta}. \quad (4)$$

An accelerometer can be approximated as a pure mass to approximately 80% of the fundamental resonant frequency of its seismic flexure. This is because even as the seismic flexure of the accelerometer approaches resonance, if properly designed, the much larger, rigid accelerometer case that houses the flexure will dominate its response. Thus, the mechanical impedance ( $Z$ ) of an accelerometer can be calculated to be:

$$Z_{\text{accel}} = (-\omega^2 m_{\text{accel}} A)/(j\omega A) = -j\omega m_{\text{accel}}. \quad (5)$$

Here,  $\omega$  is the angular frequency,  $A$  is the amplitude of displacement at  $\omega$ ,  $j = \sqrt{-1}$ , and the mass of the accelerometer is clearly identified. *Note that the impedance of the accelerometer increases with frequency.* Having made this observation, the mechanical impedance of an Endevco 7270 or PCB 3991 can easily be calculated. Two grams will approximate the mass of either accelerometer while also accounting for mounting screws and cable attachment. The magnitude of  $Z_{\text{accel}}$  then becomes  $7.178 \times 10^{-3}$  pound-seconds/inch at 100 Hz and 1.436 pound-seconds/inch at 20 KHz.

It is then desired to assess the effect of  $Z_{\text{accel}}$  on the response of the structure to which it is affixed. For a linear, elastic structure subjected to a constant harmonic forcing function, the presence of an accelerometer will modify the structural motion such that:

$$V_{\text{final}} = V_{\text{initial}} [Z_{\text{structure}}/(Z_{\text{structure}} + Z_{\text{accel}})]. \quad (6)$$

For most complex structures,  $Z_{\text{structure}}$  is undefined. Regardless, the preceding equation identifies the mechanism by which the presence of the accelerometer modifies the response of the structure.

This situation has been studied [8] for the case of an accelerometer on the end of a long, thin rod that is harmonically excited at its opposite end. This is one of the few cases where  $Z_{\text{structure}}$  can be calculated. Results are:

$$Z_{\text{structure}} = -j(\sqrt{E\rho})A\sin(\omega L/\sqrt{E/\rho}). \quad (7)$$

In the above equation, all variables remain as defined earlier with the addition of  $A$  as the cross sectional area of the rod and  $\rho$  as its material density. To perform a representative calculation, consider an aluminum rod that is 9.82 inches long with a diameter of 1.00 inches. The velocity of propagation of a longitudinal wave in this rod is  $196.4 \times 10^3$  inches/second. The 9.82 inch length was selected only because the propagation velocity ( $196.4 \times 10^3$  inches/second) is a multiple of this length allowing 20 KHz to correspond to one of the resonances of the rod. For this rod length, at 20 KHz, the magnitude of its mechanical impedance can be calculated to be 33.63 pound-seconds/inch.

If we include the effect of the accelerometer on  $V_{\text{final}}/V_{\text{initial}}$  (Eq. 6), using the values we have just calculated for the accelerometer and the rod, we determine the accelerometer will modify the response of the structure by 5%. An accelerometer of larger mass (e.g. triaxial) would produce a larger degradation. While the influence of a given model of accelerometer on the response of a structure is dependent on the accelerometer's mass, its operating frequency, and the geometry and material of the structure on which it is mounted, the result of the 20 KHz calculation (just obtained) again shows consistency as an approximate upper-measurement limit.

## CONCLUSIONS

The goal of any accelerometer measurement is to define the response of the structure to which it is affixed without disturbing the structure's motion. At high frequencies, differences between the true response of the structure and the measured response from the accelerometer will occur. These differences can be attributed to unidentified higher frequency resonances or mounting anomalies associated with the accelerometer, spatial averaging of frequencies due to the physical size of the accelerometer, and mass loading of the structure to which the accelerometer is affixed. In addition, experimental verification of the modeling of structural response of the UUT becomes increasingly limited as finite element grid size is reduced. A series of arguments has been presented that indicate a realistic high frequency limit for quantitative acceleration measurements from surface mounted accelerometers to be approximately 20 KHz. Having been presented, this limit should result in the establishment of more realistic testing goals in general and specifically assist in evolving specifications for pyrotechnic shock (pyroshock).

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